1. A student's attempt to solve the equation $2\log_2 x - \log_2 \sqrt{x} = 3$ is shown below.

$2\log_2 x - \log_2 \sqrt{x} = 3$	
$2\log_2 \left(\frac{x}{\sqrt{x}}\right) = 3$	using the subtraction law for logs
$2\log_2 \left(\sqrt{x}\right) = 3$	simplifying
$\log_2 x = 3$	using the power law for logs
$x = 3^2 = 9$	using the definition of a log

(a) Identify two errors made by this student, giving a brief explanation of each.

(2)

(b) Write out the correct solution.

(3)

a. using subtraction law for logs : the 2 in front should be the power of x

$\log_2 x^2 - \log_2 \sqrt{x} = 3$

$\log_2 \frac{x^2}{\sqrt{x}} = 3$

definition of a log: $x = 2^3$

$= 8$

b) $2\log_2 x - \log_2 \sqrt{x} = 3$

$\log_2 x^2 - \log_2 \sqrt{x} = 3(\log_2 2)$

$\log_2 \frac{x^2}{\sqrt{x}} = \log_2 2^3$

$\frac{x^2}{\sqrt{x}} = 8$

$x^{\frac{3}{2}} = 8$

$x = 8^{\frac{2}{3}}$

$= 4$

2.

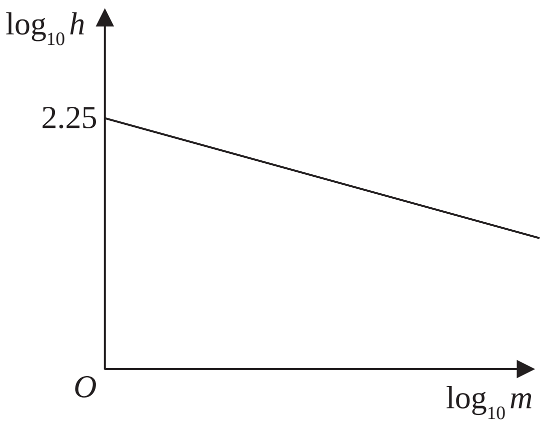

Figure 2

The resting heart rate, h, of a mammal, measured in beats per minute, is modelled by the equation

$$h = pm^q$$

where p and q are constants and m is the mass of the mammal measured in kg.

Figure 2 illustrates the linear relationship between $\log_{10} h$ and $\log_{10} m$

The line meets the vertical $\log_{10} h$ axis at 2.25 and has a gradient of -0.235

(a) Find, to 3 significant figures, the value of p and the value of q.

(3)

A particular mammal has a mass of 5 kg and a resting heart rate of 119 beats per minute.

(b) Comment on the suitability of the model for this mammal.

(3)

(c) With reference to the model, interpret the value of the constant p.

(1)

(a) Write the equation $y = mx + c$ with the given information.

$\log_{10} h = -0.235 \log_{10} m + 2.25$ — ①

Given $h = pm^q$

$\log_{10} h = \log_{10} pm^q$

$= \log_{10} p + \log_{10} m^q$

$\log_{10} h = \log_{10} p + q \log_{10} m$ ① — ② Compare ① with ②

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$\log_{10} p = 2.25$

$p = 10^{2.25}$

$= 177.8$

$p \approx 178$ (3sf) 1

$q \log_{10} m = -0.235 \log_{10} m$

$q = -0.235$ (3sf) 1

(b) $\log_{10} h = -0.235 \log_{10} 5 + 2.25$ 1

$= -0.16426 + 2.25$

$= 2.0857$

$h = 10^{2.0857}$

$= 121.8\ldots$ beats/min

≈ 122 beats/min 1

Since we're comparing this model to a heart rate of 119 beats/min, we can say that the model is reasonably accurate because if we round both values to 2sf, we will get the same value, which is 120

$\therefore$ Hence, this model is suitable to be used since it's reasonably accurate to 2sf. 1

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(c) p is the resting heart rate in beats /min of a mammal with a mass of 1 kg. 1

3. Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b-1}$$

(3)

(b) Write down the full restriction on the value of b, explaining the reason for this restriction.

(2)

a) $\log a - \log b = \log(a-b)$ (1)

using $\log a - \log b = \log\left(\frac{a}{b}\right)$

$\log\left(\frac{a}{b}\right) = \log(a-b)$

$\therefore \frac{a}{b} = a - b$

$\times b$

$a = ab - b^2$

$+ b^2$ (1)

$b^2 + a = ab$

$- a$

$b^2 = ab - a$

$b^2 = a(b-1)$

$\div (b-1)$

$a = \frac{b^2}{b-1}$ as needed (1)

b) $a = \frac{b^2}{b-1}$

$b - 1 \neq 0 \therefore b \neq 1$

Since $a > 0$ we know

Always possitive

$\frac{b^2}{b-1} > 0$ (1)

$\therefore b-1$ must be possitive because $\frac{+}{-} < 0$

so $b - 1 > 0$

$\therefore b > 1$ (1)

4. By taking logarithms of both sides, solve the equation

$$4^{3p-1} = 5^{210}$$

giving the value of p to one decimal place.

(3)

log laws

$\log(a^b) \Rightarrow b\log(a)$

$4^{3p-1} = 5^{210} \Rightarrow \log(4^{3p-1}) = \log(5^{210})$

$\Rightarrow (3p-1)\log(4) = 210\log(5)$ 1

$\Rightarrow 3p-1 = \dfrac{210\log(5)}{\log(4)}$

$\Rightarrow 3p = 243.80245 + 1$

$\Rightarrow p = 81.6008\ldots$ 1

$\Rightarrow p = \underline{81.6}$ (1 d.p) 1

5. (a) Given that

$$2\log(4-x) = \log(x+8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3)

a) $2\log(4-x) = \log(x+8)$

log laws:

$\Rightarrow \log(4-x)^2 = \log(x+8)$ ①

$a \cdot \log b = \log(b)^a$

and

$\Rightarrow (4-x)^2 = x+8$ ①

if $\log(a) = \log(b)$ then $a = b$

$\Rightarrow 16 - 8x + x^2 = x + 8$

$\Rightarrow x^2 - 9x + 8 = 0$ as required ①

(b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

(ii) State which of the roots in (b)(i) is **not** a solution of

$$2\log(4-x) = \log(x+8)$$

giving a reason for your answer.

(2)

b i) $x^2 - 9x + 8 = 0$

$\Rightarrow (x-8)(x-1) = 0$

$\Rightarrow x = 1$ and $x = 8$ ①

M	A
8	-9

-8, -1

these are our roots.

log (a) is only valid for a > 0

b ii) For $x = 8$, $2\log(4-x) = 2\log(4-8) = 2\log(-4)$; hence $x = 8$ is not valid since $2\log(-4)$ cannot be found. ①

6. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P.

Find, using algebra, the exact x coordinate of P.

(4)

$y = 3 \cdot 2^x$ and $y = 15 - 2^{x+1}$ Find point of Intersection!

$\Rightarrow \quad 3 \cdot 2^x = 15 - 2^{x+1}$ ① $* \; 2^{x+1} = 2^x \cdot 2^1 = 2^x \cdot 2$

$\Rightarrow \quad 3 \cdot 2^x = 15 - 2^x \cdot 2$

$\div 2^x$ on both sides

$\Rightarrow \quad 3 = \dfrac{15}{2^x} - 2$

$\Rightarrow \quad 5 = \dfrac{15}{2^x} \quad \Rightarrow \quad 2^x = \dfrac{15}{5}$

log laws : $\ln(a^b) = b\ln(a)$

$\Rightarrow \quad 2^x = 3$ ①

'ln' of both sides

$\Rightarrow \quad \ln(2^x) = \ln(3)$

$\Rightarrow \quad x \cdot \ln(2) = \ln(3)$ $\div \ln(2)$

$\Rightarrow \quad x = \dfrac{\ln(3)}{\ln(2)}$

② 1 mark for working
1 mark for correct answer

x coordinate of P.

7. Using the laws of logarithms, solve the equation

$$\log_3 (12y + 5) - \log_3 (1 - 3y) = 2$$

(3)

$\log_3(12y+5) - \log_3(1-3y) = 2$

log subtraction law

$\log_a(b) - \log_a(c) = \log_a\left(\frac{b}{c}\right)$

$\log_3\left(\frac{12y+5}{1-3y}\right) = 2$ ①

$\frac{12y+5}{1-3y} = 3^2$ ①

because $\log_a c = b \rightarrow a^b = c$

$12y + 5 = 9(1-3y)$

$12y + 5 = 9 - 27y$

$39y = 4$

$y = \frac{4}{39}$ ①

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8. The time, T seconds, that a pendulum takes to complete one swing is modelled by the formula

$$T = al^b$$

where l metres is the length of the pendulum and a and b are constants.

(a) Show that this relationship can be written in the form

$$\log_{10} T = b\log_{10} l + \log_{10} a$$

(2)

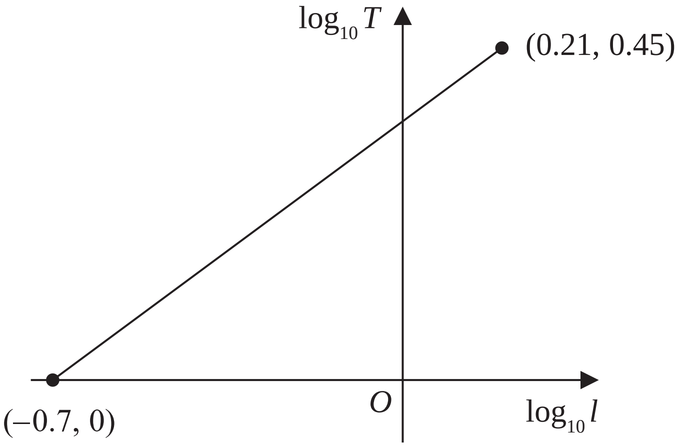

Figure 3

A student carried out an experiment to find the values of the constants a and b.

The student recorded the value of T for different values of l.

Figure 3 shows the linear relationship between $\log_{10} l$ and $\log_{10} T$ for the student's data. The straight line passes through the points (–0.7, 0) and (0.21, 0.45)

Using this information,

(b) find a complete equation for the model in the form

$$T = al^b$$

giving the value of a and the value of b, each to 3 significant figures.

(3)

(c) With reference to the model, interpret the value of the constant a.

(1)

a) $T = al^b$

$\log_{10} T = \log_{10} a + \log_{10} l^b$ ①

apply logarithmic law.

$\log_a b^c = c\log_a b$

$\log_{10} T = \log_{10} a + b\log_{10} l$ ①

Question continued

b) $\log_{10} T = \log_{10} a + b \log_{10} l$ is similar to
$y = c + mx$
therefore b is the gradient

$b = \dfrac{0.45 - 0}{0.21 - (-0.7)}$ $\quad \dfrac{y_2 - y_1}{x_2 - x_1}$

$b = 0.495$ (1)

using the point $(-0.7, 0)$

$0 = 0.495 \times -0.7 + \log_{10} a$ (1)

$0.3465 = \log_{10} a$

$a = 10^{0.3465}$ $\quad a^b = c \rightarrow \log_a c = b$

$a = 2.22$ (3.s f)

$\therefore \quad T = 2.22 l^{0.495}$ (1)
$(T = al^b)$

c) The constant a is the time taken for one swing of a pendulum of length 1m (1)